Outline

- Implementing a basic general game player
- Foundations of logic programming
- Metagaming: rule optimisation

Uses of Logic

Use logical reasoning for game play

- Computing the legality of moves
- Computing consequences of actions
- Computing goal achievement
- Computing termination

Easy to convert from logic to other representations: many orders of magnitude speedup on simulations

Things that may better be done in Logic

- Game Reformulation
- Game Analysis

Available Basic Players

Downloads - General Game Playing

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All Players Use Some Form of Logic Programming

```
true(cell(1,1,b))
...
true(cell(3,3,b))
true(control(xplayer))

legal(P,mark(M,N)) <= true(cell(M,N,b)) ^ true(control(P))
legal(xplayer,noop) <= true(control(oplayer))
legal(oplayer,noop) <= true(control(xplayer))</pre>
```

Given this logic program, answer the query

?- legal(P,M)

Substitutions

A substitution is a finite set of replacements of variables by terms

```
\{X/a, Y/f(b), V/W\}
```

The result of applying a substitution σ to an expression ϕ is the expression $\phi\sigma$ obtained from ϕ by replacing every occurrence of every variable in the substitution by its replacement.

```
p(X,X,Y,Z) \{ X/a,Y/f(b),V/W \} = p(a,a,f(b),Z)
```

Unification

A substitution σ is a <u>unifier</u> for an expression ϕ and an expression ψ if and only if $\phi\sigma=\psi\sigma$.

 $move(X,Y) \{X/a,Y/b,V/b\} = move(a,b)$ $move(a,V) \{X/a,Y/b,V/b\} = move(a,b)$

If two expressions have a unifier, they are said to be unifiable.

move(X,X) and move(a,b) not unifiable

Most General Unifiers

- A substitution σ is more general than a substitution θ if and only if there is a substitution τ such that σ ∘ τ = θ.
- A substitution σ is a most general unifier (mgu) of two expressions if and only if it is more general than any other unifier.

Theorem: If two expressions are unifiable, then they have an mgu that is unique up to variable permutation.

 $move(X,Y) \{X/a,Y/V\} = move(a,V)$ $move(a,V) \{X/a,Y/V\} = move(a,V)$ $move(X,Y) \{X/a,V/Y\} = move(a,Y)$ $move(a,V) \{X/a,V/Y\} = move(a,Y)$

Resolution

Given:		
	Query $L_1 \wedge L_2 \wedge \wedge L_m$	(without negation)
	Clauses	(without negation)
Let:		
	$A \leq B_1 \land \dots \land B_n$	"fresh" variant of a clause
	σ mgu of $L_{\rm 1}$ and A	
Then $L_1 \wedge L_2 \wedge \wedge L_m \rightarrow (B_1 \wedge \wedge B_n \wedge L_2 \wedge \wedge L_m)\sigma$		
is a <u>resolution step</u> .		

Query Answering

- A sequence of resolution steps is called a <u>derivation</u>.
- A <u>successful</u> derivation ends with the empty query.
- The <u>answer substitution</u> (computed by a successful derivation) is obtained by composing the mgu's σ₁ ∘ ... ∘ σ_n of each step (and restricting the result to the variables in the original query).
- A <u>failed</u> derivation ends with a query to which no clause applies.

Example

```
true(cell(1,1,b))
...
true(cell(3,3,b))
true(control(xplayer))

legal(P,mark(M,N)) <= true(cell(M,N,b)) ^ true(control(P))
legal(xplayer,noop) <= true(control(oplayer))
legal(oplayer,noop) <= true(control(xplayer))</pre>
```

Query ?- legal(P,M) has the following answers:

```
{P/xplayer, M/mark(1,1)}, ..., {P/xplayer, M/mark(3,3)}
{P/oplayer, M/noop}
```

Query Answering with Negation

Given:

```
\begin{array}{l} \textbf{Query} \ L_1 \ \land \ L_2 \ \land \ \ldots \ \land \ L_m \\ \textbf{Clauses} \end{array}
```

- If L₁ is an atom, proceed as before
- If L_1 is of the form $\neg A$:
 - if all derivations for A fail then $L_1 \wedge L_2 \wedge ... \wedge L_m \rightarrow L_2 \wedge ... \wedge L_m$
 - if there is a successful derivation for A then $L_1 \, \wedge \, L_2 \, \wedge \, ... \, \wedge \, L_m \to \, \text{fail}$

Example

```
role(red)
role(blue)
role(green)
true(freecell(blue))
trapped(P) <= role(P) \land ¬true(freecell(P))
goal(P,100) <= role(P) \land ¬trapped(P)</pre>
```

Query ?- goal(P,100) has the only answer {P/blue}

Query Answering with Disjunction

A clause with a disjunction

 $A <= B \land (C_1 \lor C_2) \land D$

is logically equivalent to the conjunction of the clauses

$$A \le B \land C_1 \land D$$
$$A \le B \land C_2 \land D$$

Some Rules You Don't Want to Allow

role(player(X))

```
next(control(white)) <= p
next(control(black)) <= r
p <= ¬r
r <= ¬p</pre>
```

How to Guarantee Finiteness (Part 1)

A clause is <u>safe</u> if and only if every variable in the clause appears in some positive subgoal in the body.

• Safe Rule:

 $r(X,Y) <= p(X,Y) \land q(Y,Z) \land \neg r(X,Z)$

• Unsafe Rule:

 $r(X,Z) <= p(X,Y) \land q(Y,X)$

Unsafe Rule:

 $r(X,Y) <= p(X,Y) \land \neg q(Y,Z)$

In GDL, all rules are required to be safe. (Note that this implies all facts to be variable-free.)

Dependency Graph

The <u>dependency graph</u> for a set of clauses is a directed graph in which

- the nodes are the relations mentioned in the head and bodies of the clauses
- there is an arc from a node p to a node q whenever p occurs in the body of a clause in which q is in the head.



A set of clauses is <u>recursive</u> if its dependency graph contains a cycle.

How to Guarantee Finiteness (Part 2)

A set of rules is said to be <u>stratified</u> if there is no recursive cycle in the dependency graph involving a negation.

Stratified:

 $t(X,Y) \le q(X,Y) \land \neg r(X,Y)$ $r(X,Z) \le p(X,Y)$ $r(X,Z) \le r(X,Y) \land r(Y,Z)$

Not stratified:

r(X,Z) <= p(X,Y) r(X,Z) <= p(X,Y) ∧ ¬r(Y,Z)

In GDL, all game descriptions are required to be stratified.

Logic Programming

Metagaming: Rule Optimisation

Example:

```
goal(X,Z) <= p(X,Y) \land q(Y,Z) \land distinct(Y,b)
```

Better:

```
goal(X,Z) <= p(X,Y) \land distinct(Y,b) \land q(Y,Z)
```

The argument domains can be determined from the rules of the game with the help of the dependency graph.

```
succ(0,1)
succ(1,2)
succ(2,3)
init(step(0))
next(step(X)) <=
true(step(Y)) ^
succ(Y,X)</pre>
succ(Y,X)
succ(Y,X)
```

Rule Optimisation Based on Domains

Example:

wins(P) <= true(cell(X,Y,P)) ^ corner(X,Y) ^ king(P)</pre>

Solution Set Sizes:

```
|true(cell(X,Y,P))| = 768
|corner(X,Y)| = 4
|queen(P)| = 2
```

Better Version:

```
wins(P) <= king(P) \land corner(X,Y) \land true(cell(X,Y,P))
```

Pre-Computing Answers

The ancestor relation is the transitive closure of the parent relation:

```
ancestor(X,Y) <= parent(X,Y)
ancestor(X,Z) <= ancestor(X,Y) \land ancestor(Y,Z)</pre>
```

The "samefamily" relation is true of all pairs of people that are relatives, i.e., that have a common ancestor:

```
sf(Y,Z) \ll ancestor(X,Y) \land ancestor(X,Z)
```

If we pre-compute ancestor then we increase the computational efficiency of answering the query sf.

Hitch: database storage space

Not a Good Idea to Pre-Compute sf



Logic Programming

Better: Pre-Compute ancestor



Even Better: Pre-Compute a New Relation



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Outlook: Building a Good General Game Player

- Playing Single-Player Games (a.k.a. Planning)
- Stochastic Search
- Automatic Heuristics Generation